

EUROPHYSICS LETTERS

Europhys. Lett., (), pp. ()

Gauge dependence of the order parameter anomalous dimension in the Ginzburg-Landau model and the critical fluctuations in superconductors

F. S. NOGUEIRA

Centre de Physique Théorique, Ecole Polytechnique, F-91128 Palaiseau Cedex, FRANCE

(received ; accepted)

PACS. 74.20.-z.05.10.00.-z.

Abstract. – The critical fluctuations of superconductors are discussed in a fixed dimension scaling suited to describe the type II regime. The gauge dependence of the anomalous dimension of the scalar field is established exactly from the Ward-Takahashi identities. Its fixed point value gives the η critical exponent and it is shown that η is gauge independent, as expected on physical grounds. In the scaling considered, η is found to be zero at 1-loop order, while $\nu \approx 0.63$. This result is just the 1-loop values for the XY model obtained in the fixed dimension renormalization group approach. It is shown that this XY behavior holds at all orders. The result $\eta = \eta_{XY}$ should be contrasted with the negative values frequently reported in the literature.

The high temperature superconductors have a larger critical region relative to ordinary superconductors. This fact allows in principle an experimental access to the critical region and measurements of critical exponents had been made in these materials, specially the YBCO ones [1, 2]. The experiments showed that critical fluctuations are very important in high temperature superconductors, a consequence of this fact being the non-mean-field values of the critical exponents. In the tested temperature region it was obtained that $-0.03 < \alpha < 0$ and $\nu \approx 0.67$ while the amplitude ratio $A^+/A^- = 1.065 \pm 0.01$. These values are consistent with those found for ^4He and correspond to a *uncharged* $d = 3$ XY universality class. There are some subtleties with respect to this conclusion and the interpretation of the experimental data. For instance, the value of the ν exponent has been obtained by a direct measurement of the London penetration depth, λ , which was found to scale as $\lambda \sim t^{-y}$, t being the reduced temperature and $y = 0.33 \pm 0.01$ [2]. The value of ν follows by assuming that the superfluid density scales with λ as $\rho_s \sim \lambda^{-2}$ and through the use of Josephson's relation $\rho_s \sim \xi^{-1}$ [3]. It has been argued by Herbut and Tesanovic [6] that in a superconducting transition λ should diverge with the same exponent as ξ , contradicting the relation $\rho_s \sim \lambda^{-2}$. In fact, they claim that the correct relation should $\rho_s \sim \lambda^{-1}$, when the critical fluctuations near the *charged* fixed point are taken into account. Thus, it seems to be very difficult to have an experimental access to the charged fixed point, even in the case of cuprates superconductors. The critical region probed in Refs. [1, 2] corresponds to a crossover regime where the gauge field fluctuations

are unimportant. Theoretically, the nature of the charged fixed point has been elucidated in the early eighties through the study of lattice abelian gauge models using duality arguments [4]. It has been shown that the normal-superconducting transition should be a second order phase transition, at least for type II superconductors. This means that it must exist an infrared stable charged fixed point, contradicting the weak first order transition scenario of the ϵ -expansion [10]. More recent studies performed directly in continuous models [5, 6, 7, 8] and recent numerical simulations in the lattice [9], gives further support to this view.

However, some important theoretical questions concerning the scaling in the Ginzburg-Landau (GL) model are still open. For instance, everybody agrees that the critical behavior should be the XY one, but nobody is able to find the XY value for the η exponent from renormalization group calculations. More importantly, the value generally found in these calculations is negative. Negative values of η does not violate the scaling relations provided $\eta > -1$ and, in fact, slightly negative values had been reported and this bound is fulfilled [10, 11, 12, 13, 6, 14, 8]. However, the best XY estimate is slightly positive, $\eta \approx 0.04$. Therefore, the negative values found are possibly an artifact of perturbation theory. It is worth to mention, however, that respectable values for ν consistent with the XY behavior are found in some RG calculations [5, 6, 8].

Nearly five years ago, Kiometzis and Schakel [15] argued that negative values of η , though > -1 , should be unphysical since in principle it violates unitarity in the corresponding quantum field theory. Moreover, η is the fixed point value of the anomalous dimension η_ϕ of the order parameter field in the GL model, a quantity which is gauge dependent. Based on physical intuition only we may suspect that η_ϕ should be gauge independent at the fixed point. However, there is no actual proof of this fact to date. For this reason we shall address this point in this paper by using a fixed dimension RG approach for $T > T_c$. As will be made apparent soon, the critical point fixed dimension approach employed in references [6, 8] is not well suited for the following analysis. By making use of the Ward-Takahashi (WT) identities, we shall establish the gauge dependence of η_ϕ at all orders in perturbation theory. This step follows from general field theoretical arguments [16]. Next we shall show that at the critical point η_ϕ is in fact gauge independent. Finally, we perform a 1-loop calculation to obtain $\eta = 0$, a result consistent with the 1-loop result for the XY model. We show that higher order corrections will improve further this result to obtain $\eta = \eta_{XY}$.

Our starting point is the bare action for the GL model, or Euclidean scalar QED in three dimensions,

$$S = \int d^3x \left[\frac{1}{4} F_0^2 + (D_\mu^0 \phi_0)^\dagger (D_\mu^0 \phi_0) + \frac{M_0^2}{2} A_\mu^0 A_\mu^0 + m_0^2 |\phi_0|^2 + \frac{u_0}{2} |\phi_0|^4 \right] + S_{gf}, \quad (1)$$

where the zeroes denote bare quantities, F_0^2 is a short for $F_0^{\mu\nu} F_0^{\mu\nu}$ and $D_\mu^0 = \partial_\mu + ie_0 A_\mu^0$. The S_{gf} is the gauge fixing part and is given by

$$S_{gf} = \int d^3x \frac{1}{2\alpha_0} (\partial_\mu A_\mu^0)^2. \quad (2)$$

We introduced a mass to the vector field in order to regularize the infrared divergences arising from some diagrams containing gauge fields propagators. This mass term breaks gauge invariance but as we shall see, gauge invariance is restored at the infrared stable fixed point.

The renormalized action is defined by $S' + \delta S$ where S' is the same as the bare action but with renormalized quantities while δS is the counterterm action. It is given by

$$\begin{aligned} \delta S = & \int d^3x \left[\frac{Z_A - 1}{4} F^2 + (Z_\phi - 1)(D_\mu \phi)^\dagger (D_\mu \phi) + (Z_A M_0^2 - M^2) A_\mu A_\mu \right. \\ & \left. + (Z_\phi m_0^2 - m^2) |\phi|^2 + (Z_u - 1) \frac{u}{2} |\phi|^4 + \frac{Z_\alpha - 1}{2\alpha} (\partial_\mu A_\mu)^2 \right], \end{aligned} \quad (3)$$

with the renormalized fields defined by $A_\mu = Z_A^{-1/2} A_\mu^0$ and $\phi = Z_\phi^{-1/2} \phi_0$.

By adding sources terms for the corresponding fields, it is straightforward to derive the following WT identity:

$$\left\{ \left(M^2 - \frac{1}{\alpha} \Delta \right) \partial_\mu \frac{\delta}{\delta J_\mu(x)} + ie \left[J^\dagger(x) \frac{\delta}{\delta J^\dagger(x)} - J(x) \frac{\delta}{\delta J(x)} \right] \right\} W(J_\mu, J^\dagger, J) = \partial_\mu J_\mu(x), \quad (4)$$

where $W = \log Z$, Z being the generating functional of correlation functions. When the sources are zero Z corresponds to the partition function. The W generates the connected correlation functions. The Legendre transform of W is performed as usual and gives the functional $\Gamma(\varphi^\dagger, \varphi, a_\mu)$ which is the generator of the 1-particle irreducible functions. It satisfies a WT identity which is the Legendre transform of (4):

$$\left(\frac{1}{\alpha} \Delta - M^2 \right) \partial_\mu a_\mu(x) + \partial_\mu \frac{\delta \Gamma}{\delta a_\mu(x)} + ie \left[\varphi(x) \frac{\delta \Gamma}{\delta \varphi(x)} - \varphi^\dagger(x) \frac{\delta \Gamma}{\delta \varphi^\dagger(x)} \right] = 0. \quad (5)$$

The WT identity given by Eq.(5) gives important informations about the counterterms. For example, it implies that non-gauge invariant terms in the renormalized action are not renormalized and consequently the corresponding counterterms are zero. This implies $M^2 = Z_A M_0^2$ and $\alpha = Z_A^{-1} \alpha_0$. Gauge invariance also implies $e^2 = Z_A e_0^2$. Let us define the following dimensionless gauge couplings, $\hat{e}^2 = e^2/m$ and $v = m/M$. We have the following exact flow equations:

$$m \frac{\partial M^2}{\partial m} = \eta_A M^2, \quad (6)$$

$$m \frac{\partial \alpha}{\partial m} = -\eta_A \alpha, \quad (7)$$

$$m \frac{\partial \hat{e}^2}{\partial m} = (\eta_A - 1) \hat{e}^2, \quad (8)$$

$$m \frac{\partial v}{\partial m} = \left(1 - \frac{\eta_A}{2} \right) v, \quad (9)$$

where we have introduced the RG function η_A which is the anomalous dimension of the gauge field. It is defined by

$$\eta_A = m \frac{\partial}{\partial m} \log Z_A. \quad (10)$$

An immediate consequence of (8) is that at a charged fixed point we must have $\eta_A = 1$. This very simple observation have important implications concerning the scaling of the magnetic field penetration depth, as first observed by Herbut and Tesanovic [6] (see also ref. [17]). It follows also from the above equations that at the charged fixed points the corresponding fixed point value of α is $\alpha^* = 0$, that is, the Landau gauge. Note that the situation here is

somewhat different from that one encountered in particle physics where $d = 4$. In fact, in that case the beta function for \hat{e}^2 is $\eta_A \hat{e}^2$ and, therefore, the charged fixed point would correspond to $\eta_A = 0$. Since Eq.(7) remains the same for $d = 4$, we obtain that the fixed point value of α is arbitrary in this case.

From Eq.(6) we obtain that the charged fixed point value of M^2 is zero. Therefore, near the superconducting fixed point the effective action flows to a configuration with massless gauge fields in the Landau gauge. This explains why RG calculations performed in the Landau gauge gives good results. We can say, therefore, that it is legitimate to compute critical exponents in the Landau gauge even knowing that some RG functions like $\eta_\phi = m \partial \log Z_\phi / \partial m$ are gauge dependent. This is in contrast to η_A which is gauge independent if a minimal subtraction scheme is used.

In order to check the consistency of this argument it remains to show that η_ϕ is in fact well behaved with respect to the gauge dependence as approaching the critical point. We can obtain the exact gauge dependence of η_ϕ from the WT identity Eq.(4). Indeed, we can use (4) to relate scalar 2-point correlation functions with a $(\partial_\mu A_\mu)^2$ insertion to the 2-point scalar correlation functions. This is obtained by applying twice the WT identity (4) to the 2-point correlation functions. We obtain in this way the exact equation

$$W_{(\partial_\mu A_\mu)^2}^{(2)}(p) = 2e^2 \int \frac{d^3 k}{(2\pi)^3} \frac{\alpha^2}{(k^2 + \alpha M^2)^2} \left[W^{(2)}(p+k) - W^{(2)}(p) \right], \quad (11)$$

out of which we get the *exact* gauge dependence of η_ϕ :

$$\frac{\partial \eta_\phi}{\partial \alpha} = -\frac{\eta_A \hat{e}^2 v}{8\pi \sqrt{\alpha}}, \quad (12)$$

with η_A being gauge independent. As a charged fixed point is approached, $\eta_A \rightarrow 1$, $v \rightarrow 0$, $\hat{e} \rightarrow \hat{e}^*$ and $\alpha \rightarrow 0$. In order to have a consistent η exponent it is necessary to show that $v/\sqrt{\alpha} \rightarrow 0$ as we approach the critical point. This is in fact the case since from Eqs.(7) and (9) we obtain that $v/\sqrt{\alpha}$ scales like $m = \xi^{-1}$ near the critical point. Therefore, we have $\partial \eta_\phi / \partial \alpha = 0$ at the critical point. The same result is obtained if we consider a more general model including a Chern-Simons term whose critical behavior has been studied recently [8, 18, 19]. Although in this case the photon has a massive propagator without breaking of gauge invariance, we must still keep the mass M above in order to recover consistently the non-topological model in the zero Chern-Simons mass limit. Note that the critical point approach [6, 8], though it regularizes infrared divergent graphs, is not appropriated to discuss in a complete way the gauge dependence of η_ϕ . In fact, at the critical point $m = M = 0$ and we have infrared divergences in (11).

Let us perform now a sample calculation up to 1-loop order in the Landau gauge. For this end we write $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$ and use the the renormalization conditions for the irreducible vertex functions: $\Gamma_{11}^{(2)}(0) = m^2$, $\Gamma_{\mu\mu}^{(2)} = 3M^2$, $\partial \Gamma_{11}^{(2)}(0)/\partial p^2 = 1$, $\partial \Gamma^{(2)}(0)_{\mu\mu}/\partial p^2 = 2$ and $\Gamma_{1111}^{(4)}(0) = 3u$. The corresponding anomalous dimensions are given by

$$\eta_A = \frac{\hat{e}^2}{24\pi}, \quad (13)$$

$$\eta_\phi = -\frac{2}{3\pi} \frac{\hat{e}^2 v^2}{(1+v)^2}. \quad (14)$$

The flow of the coupling $\hat{u} = u/m$ is given up to 1-loop order by

$$m \frac{\partial \hat{u}}{\partial m} = (2\eta_\phi - 1)\hat{u} + \frac{5}{8\pi}\hat{u}^2 + \frac{v}{2\pi}\hat{e}^4. \quad (15)$$

Figures 1 and 2 show the flow diagram respectively in the (u, f) ($f \equiv \hat{e}^2$) and the (f, v) planes. Fig.1 corresponds to a section $v = 0.001$ of the critical manifold. Note that for a small but nonzero v we have two charged fixed points, corresponding respectively to the tricritical and superconducting fixed points [6, 14, 8]. It is useful to compare the above calculation with other fixed dimension approaches, for instance, the one employed by Herbut and Tesanovic and de Calan *et al.* [6, 8]. In the approach of Refs. [6, 8] the charged fixed points are obtained in a critical point calculation through the introduction of a constant parameter c , giving the ratio between two different momentum scales of the problem, namely, the momentum determining the running of \hat{e}^2 and the one determining the running of \hat{u} . The parameter c can be adjusted in order to generate charged fixed points. The arbitrariness of c is removed by fixing it from a known numerical value of the Ginzburg constant, κ , at the tricritical fixed point. In our case, it is v that plays the role of c , v representing the ratio between the two existing scales in our problem, namely, m and M . An important difference between the present approach and the one of Refs. [6, 8] is that we do not need to fix numerically v since it flows naturally to a fixed point value. Note that the infrared stable fixed point in the flow diagram of Fig.2 is charged. The problem with the ϵ -expansion is that only one scale is considered. In such a model we have naturally two scales, which are of course related. For instance, the most natural scales in the broken symmetry phase are λ and ξ , whose ratio gives an important physical parameter, the Ginzburg constant κ . However, the ϵ -expansion has the advantage of being a controlled approximation, in the sense that we have a well defined small parameter. In the fixed dimension approach, a good choice of expansion parameter is $1/N$, N being the number of order parameter components. The main drawback in this case is that it is not easy to extrapolate the value of N to the physical case $N = 2$. Thus, our approximation, though uncontrolled (just like those in Refs. [6, 8, 14], enables us to get sensible physical results. Anyway, it is possible in principle to use controlled approximations like the ϵ -expansion to obtain results consistent with the existence of an infrared stable charged fixed point. This can be accomplished by considering explicitly the two scales of the model. This problem is treated more appropriately in the broken symmetry regime and will be the subject of a future publication.

Since $v = 0$ at the charged fixed point, we obtain exactly the critical behavior of the XY model. For instance, we find $\eta = 0$ and $\nu \approx 0.63$, the 1-loop values for XY model in the fixed dimension approach. Note that η is not negative in the present scaling. Higher order corrections behave in the same way, that is, all the powers of \hat{e}^2 are suppressed at the charged fixed point because they are multiplied by some function of \hat{v} which goes to zero at the charged fixed point. Note that these functions of \hat{v} will never be divergent as $\hat{v} \rightarrow 0$. This follows by simple dimensional analysis performed in the graphs containing gauge and scalar fields lines. Thus, if we compute higher order corrections we expect to approach asymptotically the best XY values estimates for η and ν .

In summary, we established exactly the gauge dependence of the scalar field anomalous dimension and showed that calculations are legitimate if performed in the Landau gauge. In the fixed dimension scaling considered, explicit calculations show that the GL model lies in fact in the XY model universality class.

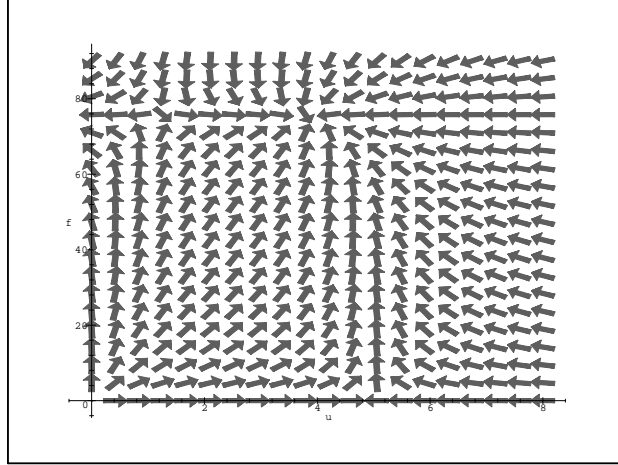


Fig. 1. – Flow diagram in the (u, f) -plane.

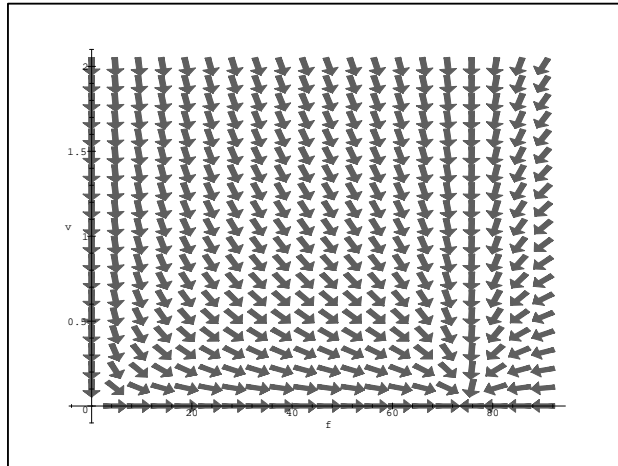


Fig. 2. – Flow diagram in the (f, v) -plane. The infrared stable fixed point is charged

The author would like to thank C. de Calan for interesting discussions. The idea of writing this paper originated from discussions with R. Folk, Yu. Holovatch, D. Loison and A.M.J. Schakel and the author is particularly indebted to them. This work was supported by the agency CNPq, a division of the Brazilian Ministry of Science and Technology.

REFERENCES

- [1] M. B. SALAMON, J. SHI, N. OVEREND and M. A. HOWSON, *Phys. Rev. B*, **47** (1993) 5520; N. OVEREND, M. A. HOWSON and I. D. LAWRIE, *Phys. Rev. Lett.*, **72** (1994) 3238.
- [2] S. KAMAL, D. A. BONN, N. GOLDENFELD, P. J. HIRSCHFELD, R. LIANG AND W. N. HARDY, *Phys. Rev. Lett.*, **73** (1994) 1845.
- [3] B. D. JOSEPHSON, *Phys. Lett.*, **21** (1966) 608.
- [4] C. DASGUPTA and B. I. HALPERIN, *Phys. Rev. Lett.*, **47** (1981) 1556; J. BARTHOLOMEW, *Phys. Rev. B*, **28** (1983) 5378.
- [5] M. KIOMETZIS, H. KLEINERT and A. M. J. SCHAKEL, *Phys. Rev. Lett.*, **73** (1994) 1975.
- [6] I. F. HERBUT and Z. TESANOVIĆ, *Phys. Rev. Lett.*, **76** (1996) 4588; I. D. LAWRIE, *Phys. Rev. Lett.*, **78** (1997) 979; I. F. HERBUT and Z. TESANOVIĆ, *Phys. Rev. Lett.*, **78** (1997) 980.
- [7] I. D. LAWRIE, *Phys. Rev. Lett.*, **79** (1997) 131.
- [8] C. DE CALAN, A. P. C. MALBOUISSON, F. S. NOGUEIRA and N. F. SVAITER, *Phys. Rev. B*, **59** (1999) 554.
- [9] P. OLSSON and S. TEITEL, *Phys. Rev. Lett.*, **80** (1998) 1964; K. KAJANTIE, M. KARJALAINEN, M. LAINE and J. PEISA, *Nucl. Phys. B*, **520** (1998) 345.
- [10] B. I. HALPERIN, T. C. LUBENSKY and S.-K. MA, *Phys. Rev. Lett.*, **32** (1974) 292; J.-H. CHEN, T. C. LUBENSKY and D. R. NELSON, *Phys. Rev. B*, **17** (1978) 4274.
- [11] I. D. LAWRIE, *Nucl. Phys. B*, **200** [FS 14] (1982) 1.
- [12] L. RADZIHOVSKY, *Europhys. Lett.*, **29** (1995) 227.
- [13] R. FOLK and Y. HOLOVATCH, *J. Phys. A*, **29** (1996) 3409.
- [14] B. BERGERHOFF, F. FREIRE, D. F. LITIM, S. LOLA and C. WETTERICH, *Phys. Rev. B*, **53** (1996) 5734.
- [15] M. KIOMETZIS and A. M. J. SCHAKEL, *Int. J. Mod. Phys. B*, **7** (1993) 4271.
- [16] J. ZINN-JUSTIN, *Quantum Field Theory and Critical Phenomena*, 2nd edition, 1993.
- [17] I. F. HERBUT, *J. Phys. A*, **30** (1997) 423.
- [18] H. KLEINERT and A. M. J. SCHAKEL,, Freie Universität preprint (1993).
- [19] A. P. C. MALBOUISSON, F. S. NOGUEIRA and N. F. SVAITER, *Europhys. Lett.*, **41** (1998) 547.